Quiz 2
Coding Theory

3^{rd} February 2006

Time: 1 hours (12:30–1:30pm)

1. Let $S = \{11010, 10111, 01010, 01101\}$. Find a basis for $C\langle S\rangle$. [5] What is the dimension k? [1] Find the binary code C. [4] Find also all cosets of C. [10]

Solution. Form and reduce our matrix A,

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Hence the basis is $\{11010, 01101, 00111\}$

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The basis has three components, therefore dimension k is 3.

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The binary code is

$$C = \{00000, 11010, 01101, 00111, 10111, 11101, 01010, 10000\}$$

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	cofactor	$\to$	*words*
I	$00000 + C$	$\to$	00000, 11010, 01101, 00111, 10111, 11101, 01010, 10000
II	$00001 + C$	$\to$	00001, 11010, 01100, 00110, 10110, 11100, 01011, 10001
III	$00010 + C$	$\to$	00010, 11000, 01111, 00101, 10101, 11111, 01000, 10010
IV	$00100 + C$	$\to$	00100, 11110, 01001, 00011, 10011, 11001, 01110, 10100

Since there other coset leaders all give one of these four cosets, therefore the number of cosets is four and all of them are listed above.

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2. Based on te factorisation $x^6 - 1 = (1+x)^2\left(1+x+x^2\right)^2$, find a binary $[6,3]$ cyclic code. [10]

Solution. Here we are given $k = 3$ and $n = 6$. List all nine monic divisors of $x^6 - 1$ and note the degree k, that is the number of different bases, of each. This is $x^i(\cdot)$, $i < k$.

$$\begin{array}{lcl}
1 & \to & k = 6 \\
1 + x & \to & k = 6 \\
1 + x + x^2 & \to & k = 5 \\
(1+x)^2 & \to & k = 4 \\
(1+x)\left(1+x+x^2\right) & \to & k = 3 \\
(1+x)^2\left(1+x+x^2\right) & \to & k = 2 \\
\left(1+x+x^2\right)^2 & \to & k = 2 \\
(1+x)\left(1+x+x^2\right) & \to & k = 1 \\
\left(1+x^6\right) & \to & k = 0
\end{array}$$

For $k = 3$;

$$(1+x)\left(1+x+x^2\right) = 1 + x^3$$

$$\begin{array}{lcl}
0 \cdot \left(1+x^3\right) & \to & 000000 \\
1 \cdot \left(1+x^3\right) & \to & 100100 \\
x \cdot \left(1+x^3\right) & \to & 010010 \\
x^2 \cdot \left(1+x^3\right) & \to & 001001
\end{array}$$

And the pairwise additions among these give us the remaining code words. Then,

$$C = \{000000, 100100, 010010, 001001, 110110, 101101, 011011, 111111\}$$

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